General robust Farkas lemmas with applications to robust optimization problems

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Abstract.

Let X, Y be locally convex Hausdorff topological vector spaces, $\mathcal{U} \neq \emptyset$ be an uncertainty set. Let further $F_u : X \times Y \to \overline{\mathbb{R}}$ be a proper function for each $u \in \mathcal{U}$ and $h : X \to \overline{\mathbb{R}}$ is a lower semicontinuous proper convex function. The report concerns characterizations (equivalence forms) of a robust inequality of the form:

$$\sup_{u \in \mathcal{U}} F_u(\cdot, 0_Y) \geq h. \tag{0.1}$$

Firstly, characterizations of general robust inequality (0.1) are proved in terms of abstract perturbational duality. Secondly, by specifying to some concrete classes of functions F_u, we get variants of characterizations of robust functional inequalities which, in different settings, give rise to generalized robust Farkas lemmas for general robust non-cpnvex, robust DC (difference of convex), robust convex, and robust linear systems in infinite dimensional spaces under some weak qualification conditions concerning epigraphs of the conjugate functions F_u^*. Many of the results on robust Farkas lemmas obtained are new while the others cover or extend the known ones in the literature.

As an application of the results obtained, some are then applied to get strong Fenchel duality and stable-strong/strong Lagrange duality for classes of robust DC and robust convex optimization problems.

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